- **R1: For a nonsymmetric matrix ...** We will clarify from the start that our method is designed for symmetric matrices. 1
- **R1:** PI is unstable if two eigenvalues are close. So what is the advantage of PI vs the analytic solution? The 2
- instability of PI is due to inaccurate initialization of the eigenvectors. In our case, the forward pass provides accurate 3
- values using SVD and PI then becomes stable during the backward pass. Another advantage over the analytic solution 4
- what we demonstrate is that PI yields an upper bound on the gradients' magnitude, which guarantees they will not 5 **explode.** As shown in Sec. 2.2, PI is the geometric expansion of the analytic solution: With $\lambda_i \geq \lambda_j$, the term $1/(\lambda_i - \lambda_j)$,
- 6
- which appears in the analytic solution, is approximated by $1/\lambda_i + 1/\lambda_i (\lambda_j/\lambda_i) + 1/\lambda_i (\lambda_j/\lambda_i)^2 + \dots + 1/\lambda_i (\lambda_j/\lambda_i)^{K-1} \le K/\lambda_i$ in PI, where K represents the power iteration number. When $\lambda_i = \lambda_j$, the term $1/(\lambda_i \lambda_j)$ in the analytic solution 7
- 8 explodes, while the PI gradients are naturally upper bounded by K/λ_i . 9
- **R1:** A hidden regularizer, ϵ , is introduced. What if you set ϵ to 0 or 10^{-12} . ϵ is commonly used to stabilize the 10 11
- computation (*e.g.*, [2] [3] & [r1]). As shown in Eq. 13, ϵ controls the gradients' upper bound and appears in the denominator. Thus, too small an epsilon, *e.g.*, 10^{-12} , will yield a large upper bound and increase the risk of gradient 12
- explosion. Following standard practice, e.g., [3], ϵ is set to 10^{-4} for all the methods, including SVD and PI, but our 13
- method always achieves 100% success rate while the others do not. 14
- **R1: Your method uses some tricks. The comparison seems unfair, since you can also truncate SVD.** For a fair 15
- comparison, we recomputed results using the same tricks to truncate the eigenvalues for our SVD baseline. For PI, 16
- truncation was already used in our submission to mitigate the round-off errors caused by the deflation process. The 17
- results on CIFAR10 are given in the table below. Note that, for matrix dimensions d>16, SVD and PI still fail in all 18
- cases. By contrast, we achieve 100% success rate even when the dimension is as large as 128. 19

Methods	Evaluation Metrics	d = 4	d = 8	d = 16	d = 32	d = 64	d = 128
SVD	Min Error & Suc. Rate	4.50%, 60%	4.75%, 33.3%	4.65%, 40%	-,0%	-,0%	-,0%
PI	Min Error & Suc. Rate	4.44%, 100 %	6.28%, 6.7%	-,0%	-,0%	-,0%	-,0%
Ours	Min Error & Suc. Rate	4.59%, 100 %	4.43%, 100 %	4.40%, 100%	4.46%, 100 %	4.44%, 100 %	4.75%, 100 %

R1: Results on CIFAR100 are far away from SOTA performance. It would be nice to compare with [r1]. Our 20 paper focuses on solving the stability issues of ED, not designing better normalization layers (i.e., PCA & ZCA). 21 Stability is measured as the success rate of the methods, which, for our purpose, is more important than accuracy. ZCA 22

and PCA constitute two applications of our method to demonstrate stability. We therefore just used simple backbones 23

(*i.e.*, ResNet18/50), which translates to accuracies inferior to the SOTA. By contrast, [r1] focuses on designing a better 24

normalization layer using an iterative normalization method. We nonetheless acknowledge the relevance of this paper, 25

which we will cite in the final version. Note that [r1] was not published at the time of NeurIPS submission. 26

R1: I doubt ED is widely used as there are numerical issues. Justify why ED is important for deep learning. 27

Indeed, ED has many numerical stability issues, and this is exactly what our paper addresses. Nevertheless, as discussed 28

in the introduction from Line 14 to 18, ED has been used for image/point matching [6,7,8], second-oder pooling [4], 29

and pose estimation [9]. It has not been well integrated into deep networks because of the numerical instability, and, as 30

stated by R3, one can expect that our paper will bring insight to this problem and be the basis for many new ideas. 31

R2: Will the failure cases become more or less common when matrices are large enough? As shown in the table 32 above, the baselines' failures become more common as d increases, whereas our method succeeds 100% of the time for 33 all dimensions. This remains true when increasing d to 128 (twice as many as in the submission), by putting the ZCA 34 layer on top of a 128-channel conv. layer. The underlying reasons are that, thanks to our use of SVD in the forward pass, 35

36 we have more accurate eigen value/vector estimates than the PI baseline, and that, as shown in Eq. 13, the gradients of

37 our method are always bounded regardless of matrix size while those of the SVD baseline may easily explode.

R2: The convergence behavior looks similar whether using the existing methods and the proposed one. 38 The

convergence curves shown in Fig. 2 are based solely on the successful cases for the baselines and ignore the failure cases 39

(see success rates in Table 2). Including these numerous failures would render the baseline curves entirely meaningless. 40

R2: In Tables 3 and 4, the prediction error is not monotone with the matrix size. For ZCA in Table 3, with 41

ResNet18, all values are virtually the same, and with ResNet50, the trend shows that larger d values, which only our 42

method can handle, give better results. For PCA in Table 4 (a,b), when too much information is preserved, some noise 43

is kept and the accuracy drops. Conversely, when too much information is removed, some useful signal is discarded and 44 the accuracy also drops. The right number of dimensions to preserve is dataset dependent and can be determined by

45 cross validation. 46

R3: Do we have to do blocks of d? Dividing the features into blocks of dimension d is only useful to compare our 47 approach with the baselines, which only succeed for small matrix dimensions. Given our stabilized ED method, the 48 blocks become unnecessary, and the largest dimension d in each experiment corresponds to not using blocks. 49

R3: PCA denoising doesn't look better than batch normalization (BN). While PCA denoising indeed has 50

marginal improvement over BN, it is not our main focus. Similarly to ZCA, PCA denoising only is another ap-51

plication to demonstrate the stability of our method. The baselines to truly look at in Table 5 are PCA(PI) and 52

PCA(SVD), which often break down in the training phase. We will emphasize this in the final version. 53

R3: Is there a way of minimizing the dependency on ZCA? We will minimize the emphasis on ZCA whitening 54 and PCA denoising in the abstract and introduction. 55